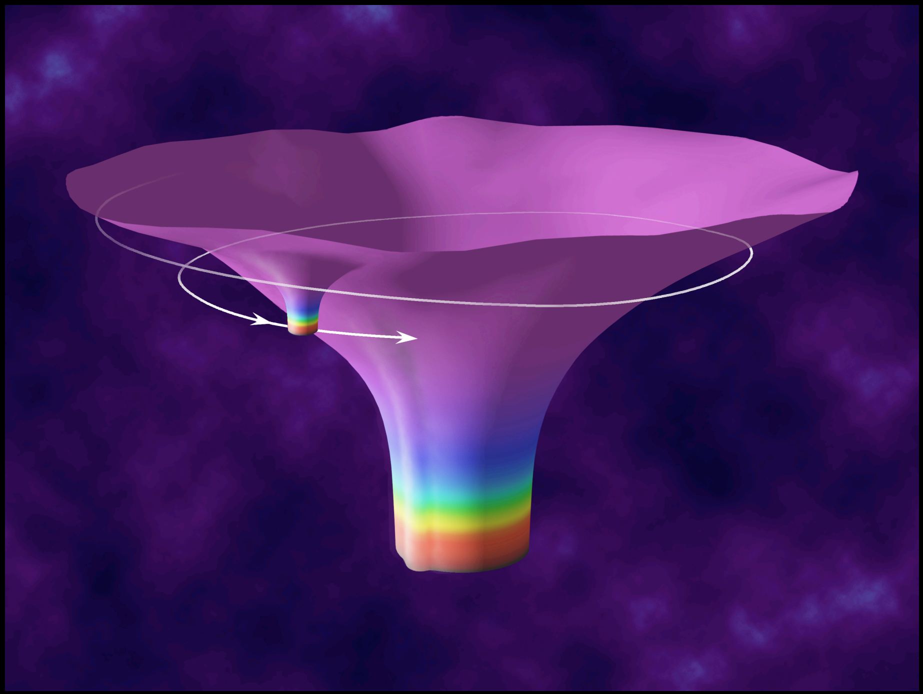
"testing relativity" with extreme mass ratio inspirals

Using extreme mass ratio capture binaries for high precision mapping of the spacetimes of massive objects in galactic nuclei



Are we really "testing relativity"?

Strictly speaking, *no*.

To be a proper test of general relativity, the "straw man" objects to which we compare our measured waveforms would have to be constructed in a framework other than general relativity.

Throughout this talk, general relativity will be *assumed* to describe the nuclear objects into which inspiral occurs.

$$\mathcal{L}_{\mathrm{EH}} = \int d^4x \sqrt{-g}\, R$$

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Will *not* examine objects in scalar tensor theories...

$$\mathcal{L}_{\rm ST} = \int d^4x\, f(\phi)\sqrt{-g}R$$

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To be a proper test of general relativity, the "straw man" objects to which we compare our measured waveforms would have to be constructed in a framework other than general relativity.

… nor in theories with different curvature terms in the action (as examples).

$$\mathcal{L}_{\rm HC} = \int d^4x\sqrt{-g}(R + \alpha R^n)$$

Are we really "testing relativity"?

My view: Remain agnostic about testing relativity!

The goal of the work discussed here:
To make precision probes of the spacetimes of massive compact objects.

Conservative hypothesis:
These bodies are black holes!

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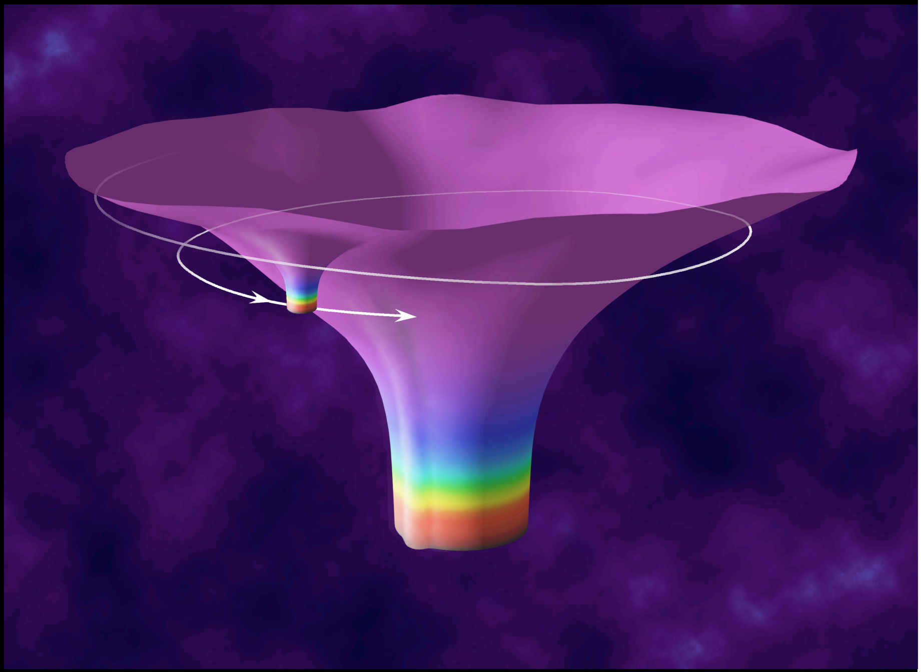
If we falsify this hypothesis, *then* we have a data-motivated reason to ponder the cause of the violation - has GR broken down? Are these just really bizarre relativistic objects?

using extreme mass ratio inspirals to test the nature of massive compact objects

Using extreme mass ratio capture binaries for high precision mapping of the spacetimes of massive objects in galactic nuclei



Sterl Phinney's top 7 list ...

1. Luminous, rapidly variable: $\epsilon > (L/t_{var})\sigma_t/(4\pi m_p c^4)$. $\epsilon > 0.01 \rightarrow$ BH accretion, not nuclear.

2. Lifetimes t so $Lt/Mc^2 > 0.01 \rightarrow$ accretion, not nuclear.

3. Jets with relativistic speeds $\rightarrow$ relativistic potential well.

4. X-ray Fe K-shell lines —redshifts and Doppler boosts imply orbits at $v \sim 0.5c$.

5. Rapid large amplitude variability implies small size $R \sim c\Delta t$. Mass from dynamics, or luminosity: radiation pressure limits mass $M > L\sigma_T/(4\pi G M m_p c)$. Combine: $GM/R \sim c^2$.

6. Peak L_{bb} of νL_ν at ν_{bb} defines $T_{bb} \sim h\nu_{bb}/(3k)$. Black body inner disk radius $\sim [L_{bb}/2\pi\sigma T_{bb}^4)]^{1/2}$ is few GM/c^2.

7. Some low luminosity sources seem to have $L \ll 0.01\dot{M}c^2$. No hard surface $\rightarrow$ horizon?

Sterl Phinney's rebuttal:

If you had one good reason,

You wouldn't need 7.

(Paraphrasing Richard Feynman.)

Most convincing evidence:

Observations that map "gravitational potential" deep in the strong field of black hole candidates.

If we can build such a map, we can compare to the predictions for black holes ... verify their "black holey-ness" or lack thereof.

Ideal tool: Orbital kinematics

When we do such an analysis with Earth orbits, we get ***geodesy***.

Expand Earth's gravitational potential in spherical harmonics.

$$\Phi = -\frac{GM}{r} + \frac{GM}{R}\sum_{lm}\left(\frac{R}{r}\right)^{l+1} B_{lm} Y_{lm}(\theta, \phi)$$

B coefficients determine "shape" of the potential.

Satellite orbits probe shape – maps to matter distribution of earth!

GRACE and CHAMP missions: Doing this with high precision.

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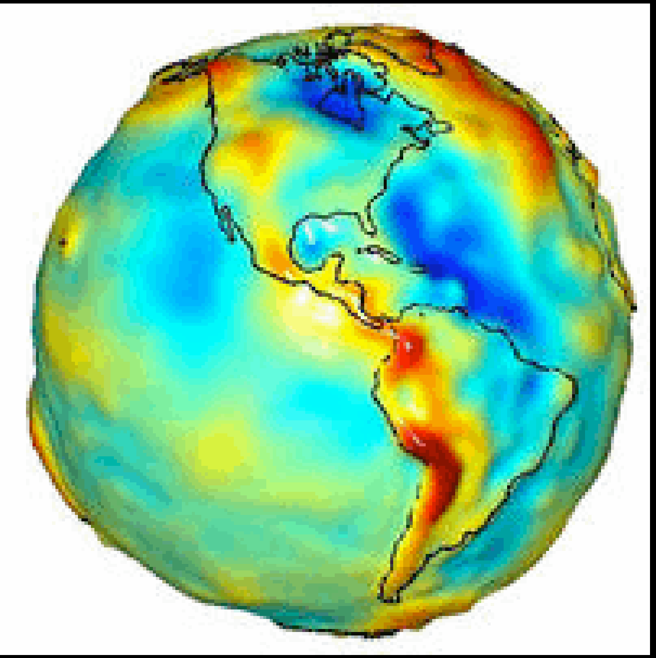

Bothrodesy: Geodesy for black holes

Bothros (βοθροσ): ancient Greek for sacrificial pit.

Same basic idea: "map" the spacetime of massive compact object. Particularly powerful test of black hole hypothesis: black holes have *very* special multipole moment structure.

Spacetime can be built from "mass moments" *M* and "current moments" *S*: $M_l + iS_l = M(ia)^l$

Only TWO moments are independent!!

Measure more than two: Have enough information to falsify the black hole hypothesis.

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Morally equivalent to multimode spectroscopy analysis presented by E. Berti: In both cases, the goal is to *overdetermine* the parameter space to check consistency with Kerr solution.

Tools of the trade

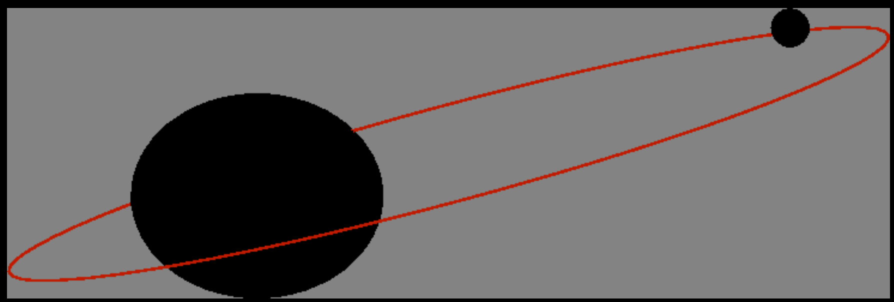
Extreme mass ratio captures.

Gory details: Following talk by Clovis Hopman.

Cartoon level details: Binary formed when many body process scatter a stellar mass compact object into an eccentric, highly relativistic orbit of the nuclear "black hole":

Zoom on

nucleus



Tools of the trade

GWs generated by these binaries in LISA band if the larger black hole is ~10^6 Msun.

Smaller member most likely a stellar mass (~10 Msun) black hole (mass segregation) - signal is detectable out to z ~ 1.

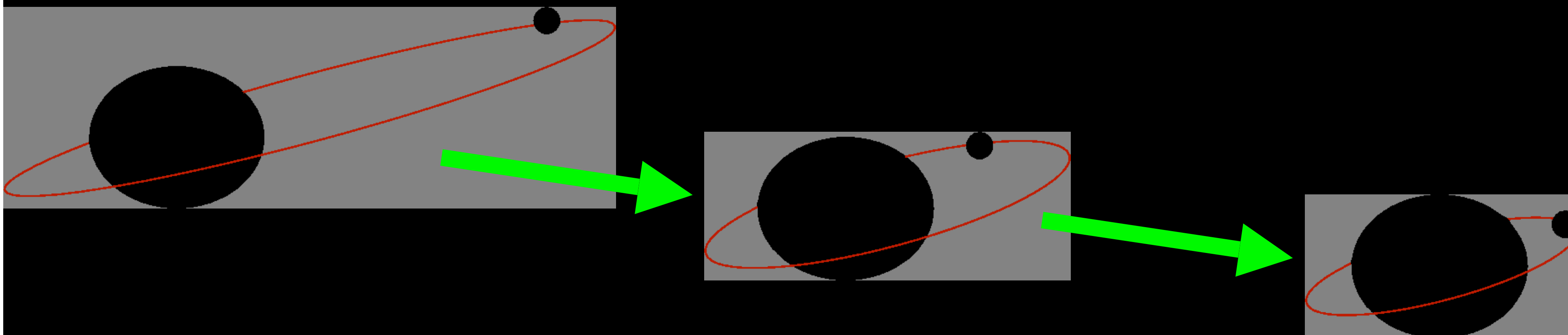
Consequence: Measured event rate could be quite high! Estimates suggest could reach 100s or even 1000s of events per year.

Possibly too much of a good thing:

Source *could* be (mildly) confusion limited (cf. Barack & Cutler 2004)

Tools of the trade

Gravitational wave emission drives the orbit to simultaneously shrink and circularize:



Detailed calculation of these GWs and their effect on orbit evolution requires strong field radiation reaction analysis.

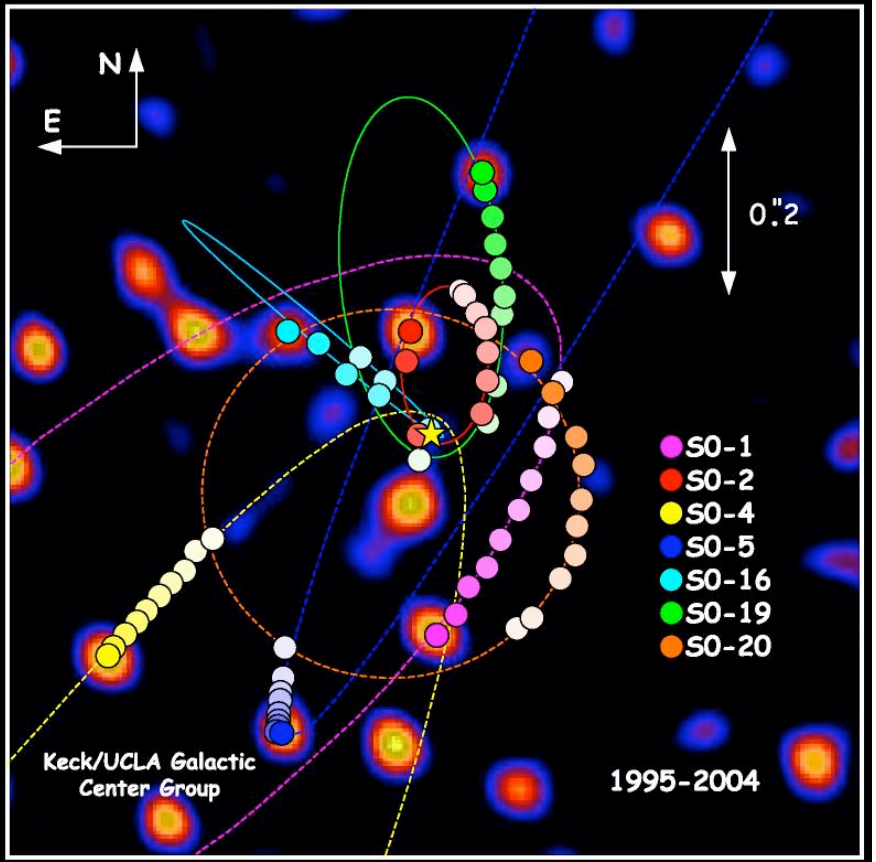
Last ~10^5 orbits generate LISA band waves from the deep strong field (r ~ a few x M)

Early holiodesy...

Andrea Ghez (UCLA) tracking orbits of stars in the Milky Way's center.

Apply Kepler – measure the mass of our black hole with < 10% error!

Not sufficiently strong field to get other multipoles?



Nice idea ... how do we **actually** extract these multipoles?

We need a theoretical framework for extracting the multipoles ... which means we need to understand objects with “arbitrary” multipole moments.

Extremely difficult problem!

Past work by Fintan Ryan: Studied GW emission and inspiral in the spacetime of an axisymmetric object with arbitrary multipoles.

Beautiful proof of principle! ***But***, weak-field calculation – very difficult to generalize.

Nice idea ... how do we **actually** extract these multipoles?

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Extremely difficult problem!

Past work by Glampedakis & Babak; Kesden, Gair & Kamionkowski: Study GW emission and inspiral in the spacetime of boson star.

Strong field, exact spacetime; plausible alternative to a black hole. But not *generic*.

What if object *IS a black hole?*

These approaches do not incorporate the black hole limit!

Arbitrary multipoles: Technically includes this limit, but requires poorly convergent infinite sums to handle strong field. Not very useful for designing a practical measurement formalism!

Boson stars: Don't include black hole limit at all! If the real objects actually *are* black holes, this straw man will totally miss it by construction.

Reformulate the question

If the massive compact objects seen in the cores of galaxies *are* black holes, their deviation from "black holey-ness" is **ZERO**.

Formulate black hole testing as a ***null experiment***: Generalize their spacetimes so that "normal" black holes corresponds to a parameter going to zero.

[Collins & Hughes, PRD **69**, 124022 (2004)]

$$
\begin{aligned}
ds^2 &= -e^{2\psi}\left(1-\frac{2M}{r}\right)dt^2 + e^{2\gamma-2\psi}\left(1-\frac{2M}{r}\right)^{-1}dr^2 \\
&\quad + r^2 e^{2\gamma-2\psi}d\theta^2 + r^2\sin^2\theta e^{-2\psi}d\phi^2 \, .
\end{aligned}
$$

New objects: **Bumpy black holes!**

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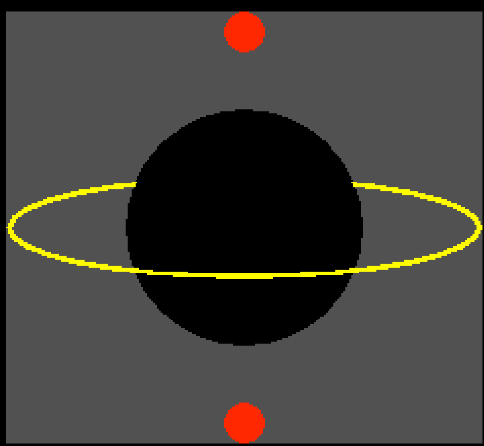
Similar construction, but more generic, developed by Glampedakis & Babak (gr-qc/0510057): "Quasi-Kerr objects"

Physical picture of a bumpy black hole

A little wierd: Object is an ordinary black hole with perturbing matter that distorts the horizon and the spacetime.



For "positive bumpiness", matter consists of ***negative*** mass (!) points at the pole and a positive mass ring around equator. Total perturbation mass is zero.

Purely quadrupolar perturbation!!

Even odder: **The extra matter corresponds to naked singularities!!!**

Makes sense ... in an odd way.

A black hole's multipole moments,

$$M_l + iS_l = M(ia)^l$$

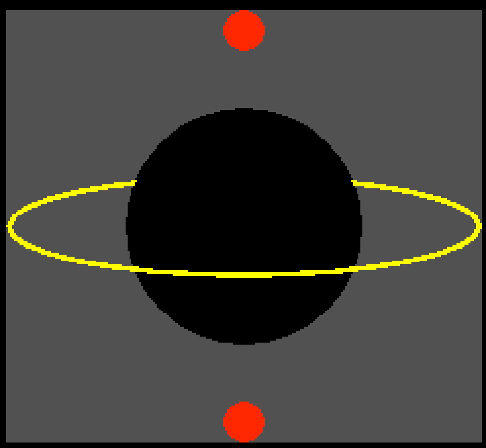
essentially states the no-hair theorem: Deviations from these canonical multipoles should radiate away quickly (Price's theorem).

Any object that is *not quite* a black hole has to be rather bizarre! Some mechanism must prevent bumps from radiating.

Guaranteed these solutions aren't physical!

Should this bother us???

NO!! Goal is not to produce a spacetime that might actually exist in nature!



Only want to provide a well-behaved *falsifiable* straw man with which to test the black hole hypothesis.

In this context, it is irrelevant whether deviations from "black holey-ness" come from reasonable physics!

Our goal is quite modest: Just proposing a framework for setting limits on any deviations which *might* exist.

Meaning of pure multipoles in GR

Point made by Saul Teukolsky:

The idea that a "pure" multipole is not necessarily physically reasonable should not bother us!

Same issue applies to electromagnetic poles - when we model an object as a sum over moments we don't actually imagine that we have infinitesimal loops of infinite current.

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GR even more pathological: Nonlinear!

Status

Currently, we can

1. Build spacetimes for some interesting but simple “bumpy black holes”.
2. Study orbits in these spacetimes.
3. Study wave emission for orbits around black holes (***NOT*** around more generic objects!)

   Caution … not fully understood in important details!
4. Use these waves to study how well we can measure black hole parameters.

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Currently, we *CANNOT*

1. Compute wave emission and inspiral in spacetimes of bumpy/quasi black holes
2. Estimate how accurately we can constrain the black hole nature of black hole candidates.

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Currently, we *CANNOT*

1. Compute wave emission and inspiral in spacetimes of bumpy/quasi black holes (at least not rigorously)
2. Estimate how accurately we can constrain the black hole nature of black hole candidates (at least not rigorously).

Remainder of talk: Assume larger object *is* a black hole. How well can we calculate GWs in this (relatively) simple case?

Radiation reaction around black holes

Bad news: computing radiation for the case of interest requires a formalism that is good for strong field, fast motion. Usual approximations not so good in this regime.

Good news: Large mass ratio of the system makes different approximation possible! Treat mass ratio as a parameter governing perturbation expansion.

$$g_{\alpha\beta}^{\mathrm{Bin}} = g_{\alpha\beta}^{\mathrm{Kerr}}(M, a) + b_{\alpha\beta}(\mu)$$

$$||b_{\alpha\beta}||/||g_{\alpha\beta}^{\mathrm{Kerr}}|| \sim \mathcal{O}(\mu/M)$$

Radiation reaction around black holes

Motion in this spacetime looks like an "ordinary" geodesic orbit, supplemented with a *self force* - manifestation of the small body's interaction with *its own* distortion to the spacetime.

Schematically, motion uses "forced geodesic":

$$\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha{}_{\beta\gamma}\frac{dx^\beta}{d\tau}\frac{dx^\gamma}{d\tau} = f^\alpha$$

Big challenge: Compute the force f^α!

See papers by Mino, Sasaki, & Tanaka (PRD 1997); Quinn & Wald (PRD 1997); Poisson (LivRevRel 2003).

Adiabatic approximation

Great simplification worked out by Yasushi Mino: *If* it is possible to introduce an "adiabatic approximation", the gory self-force calculation reduces to something *much* more tractable.

$$f^\alpha = \frac{1}{2}\left(f^\alpha_{\rm ret} - f^\alpha_{\rm adv}\right)$$

"Advanced" and "retarded" contributions simple to calculate … ordinarily, we would require an integral over the past worldline of the orbiting body. Adiabaticity turns that integral into a simple averaging procedure.

Adiabatic approximation

Simple limit: Mostly just a separation of timescales. Intuitively, require that the rates of change of all orbital quantities over a single "orbit" be much smaller than the value of that quantity:

$$\dot{\chi} T_{\rm orb} \ll \chi$$

χ is any relevant orbit parameter - E, L_z, etc.

Generic orbits - inclined and eccentric - a bit tricky to define "single orbit" as there are three distinct timescales … but tractable due to some nice Fourier tricks (Schmidt 2002; Drasco & Hughes 2004).

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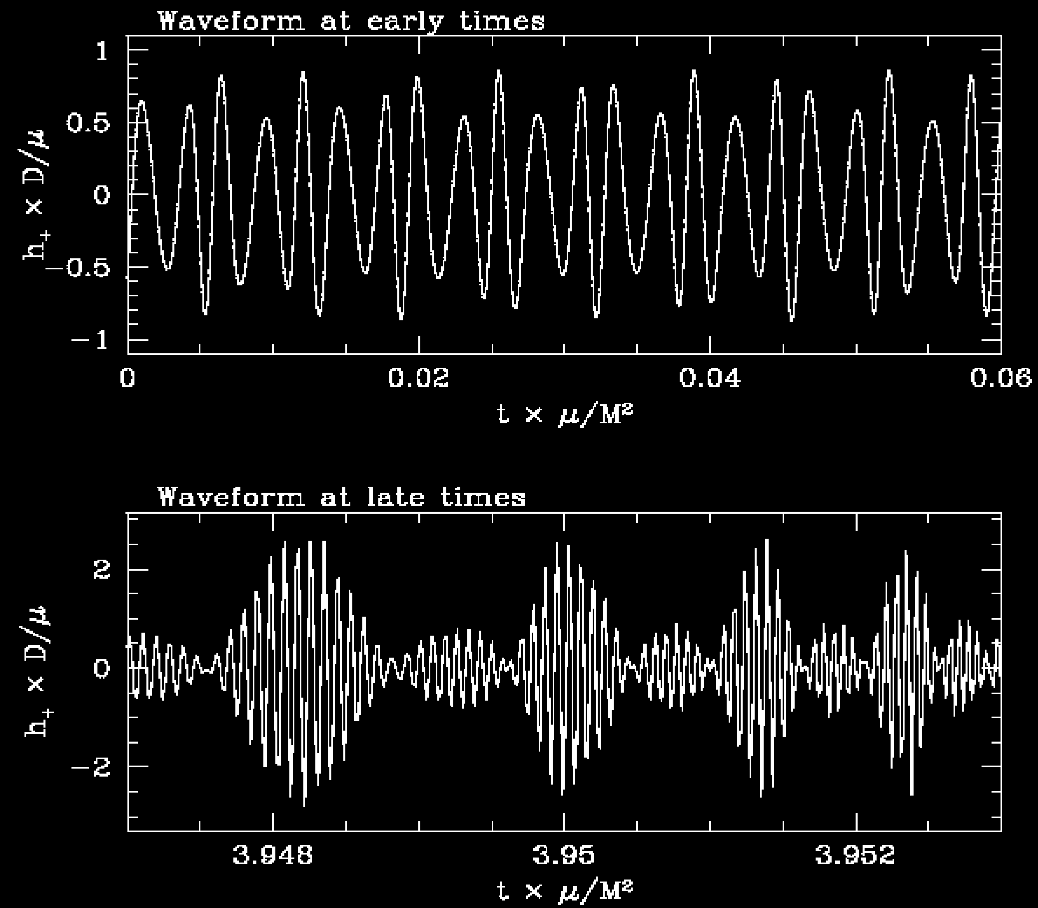
CAUTION: Throws out "conservative" effects! Systematic error must accrue (Pound, Poisson, Nickel 2005) … not certain whether this is critical (Hinderer & Flanagan in prep; Favata, in prep).

Example: Imprint of BH spin

Mismatch between θ and ϕ frequencies cause (in weak field) precession of the orbital plane (Lense-Thirring precession).

Shown: Gravitational wave snapshots for quasi-circular inspiral.

Harmonics of ϕ and θ frequencies influence this waveform.



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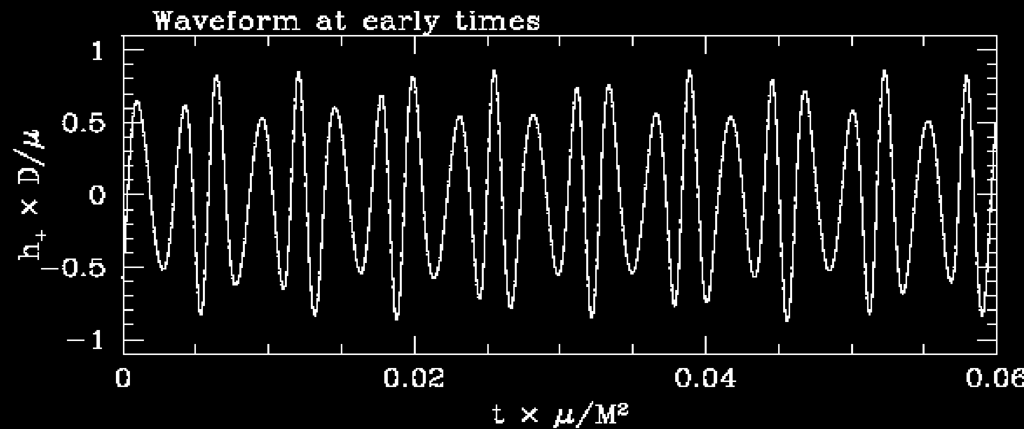
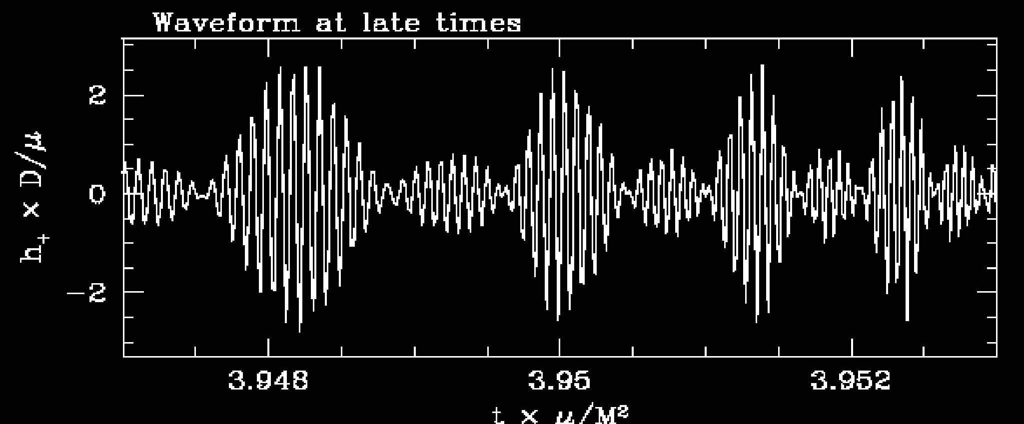
Ratio of these frequencies and their evolution provides a *strong* constraint on the nature of the massive body into which the small guy spirals!





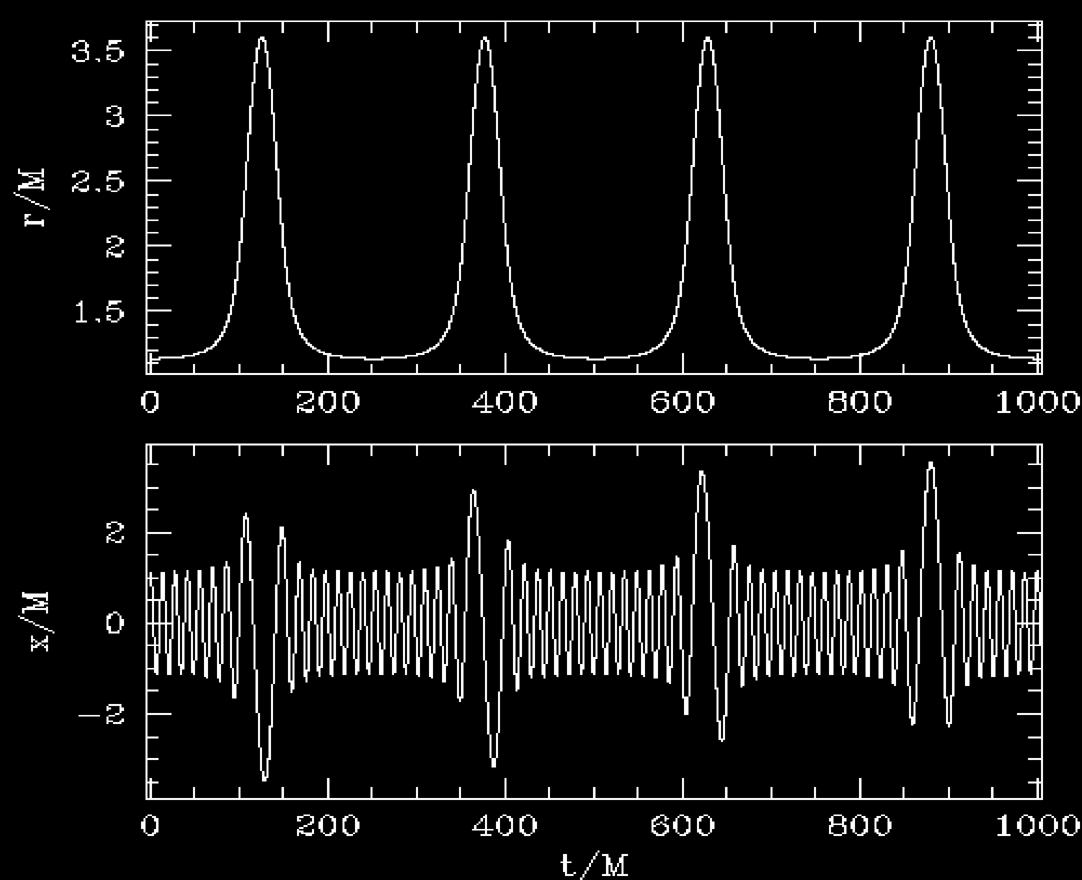
Example: Imprint of BH spin

Mismatch between ϕ and r frequencies very well known: Causes perihelion precession of Mercury. Extra azimuth is 43 arcseconds per century.

Much stronger for black hole orbits

Extra angle: thousands of radians per orbit!

Stronger gravity; intense spin-induced frame dragging.



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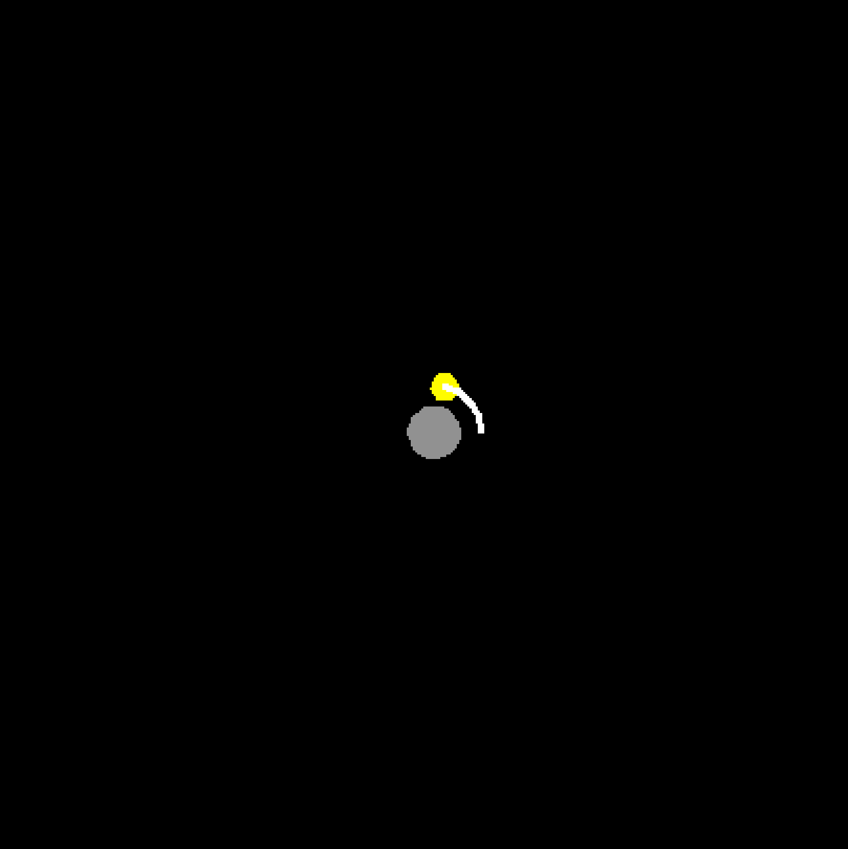

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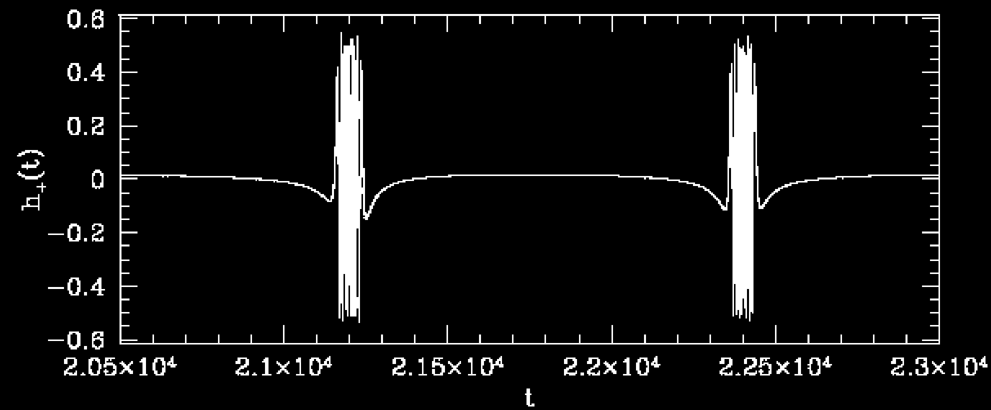

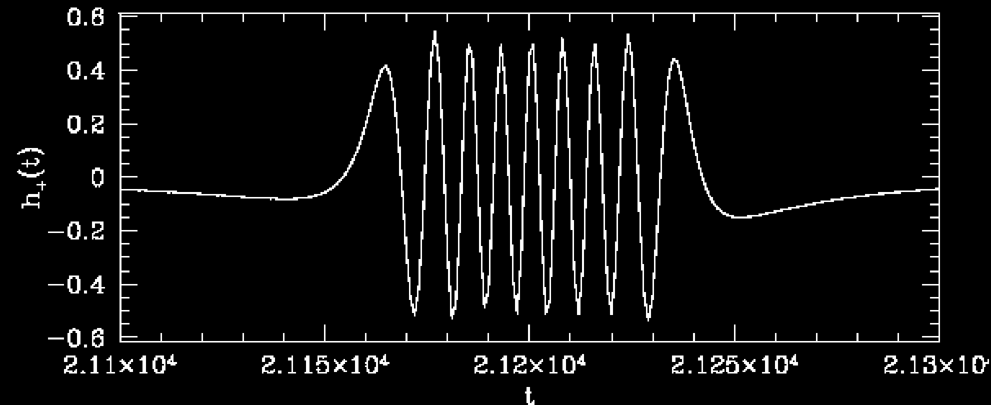

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Computing these waveforms is rather computationally expensive - cannot currently generate enough of them to explore issues in LISA data analysis.

Simpler approximations ("kludges") capture many of the features of these waveforms, but are tractable.

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Analytic kludge: Developed by Barack & Cutler, based on post-Newtonian expansion (with additional features)

Find BH mass and spin measured to about 0.01% for most favored EMRI source! (10 Msun into 10^6 Msun.)

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Computing these waveforms is rather computationally expensive - cannot currently generate enough of them to explore issues in LISA data analysis.

Simpler approximations ("kludges") capture many of the features of these waveforms, but are tractable.

Approach similar to analytic kludge used by Fintan Ryan to estimate how well multipoles can be measured for an "arbitrary" object.

$dM/M \sim 10^{-4}$

$da \sim 10^{-3}$

$d(\text{quad})/(\text{quad}) \sim 10^{-3}$

$d(\text{oct})/(\text{oct}) \sim 10^{-2}$

etc … could get out to $l \sim 5$ or 6.

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Computing these waveforms is rather computationally expensive - cannot currently generate enough of them to explore issues in LISA data analysis.

Simpler approximations ("kludges") capture many of the features of these waveforms, but are tractable.

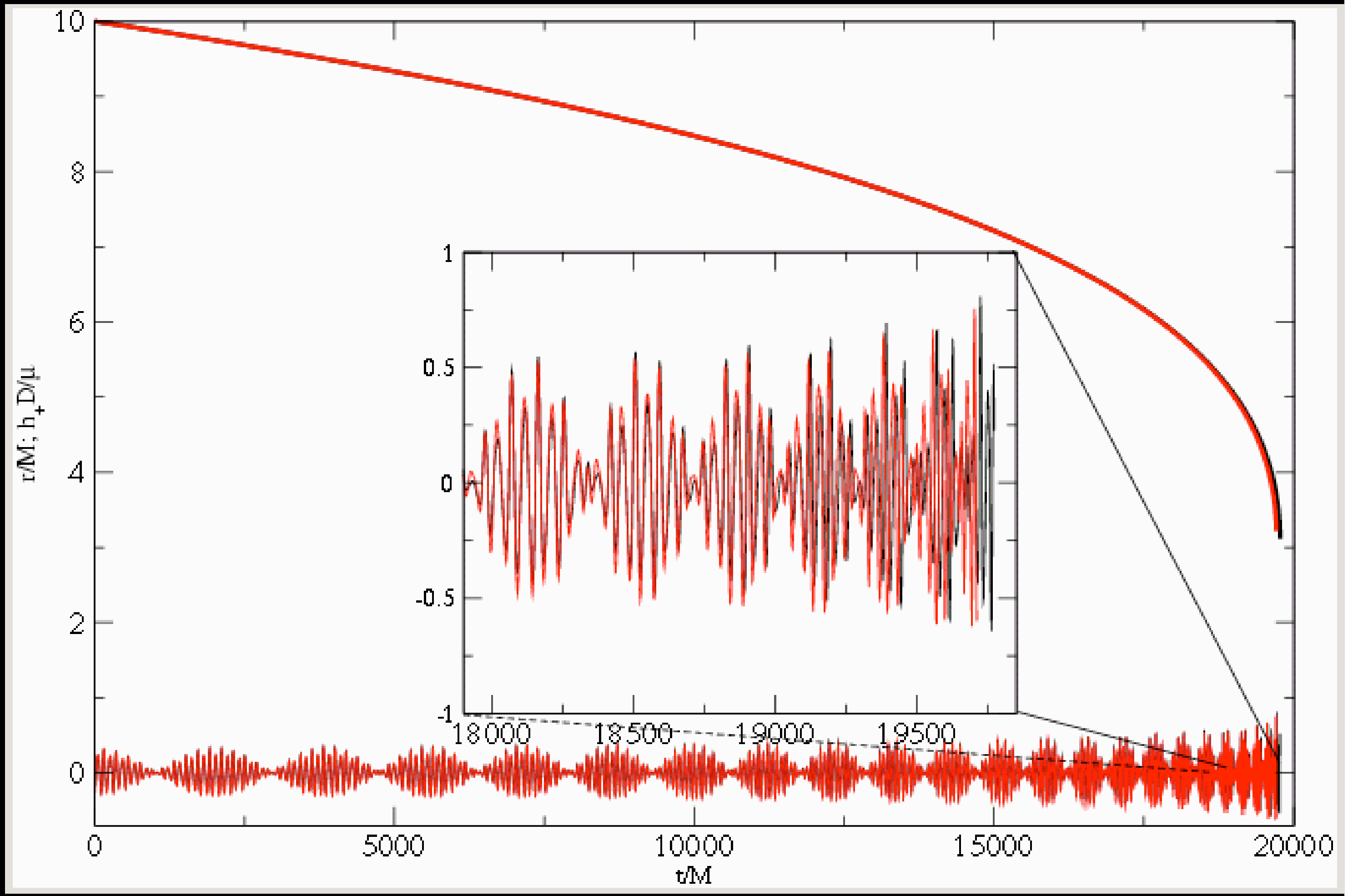
Numerical kludge: Developed by Babak et al, based on exact Kerr orbits plus an approximate description of radiation emission.

Find waveforms that agree *embarrassingly* well with strong field calculations!

Numerical kludge result

Black curve: Strong field radiation reaction. Several hundred cpu hours of computation.

Red curve: Numerical kludge. Several minutes of computation.



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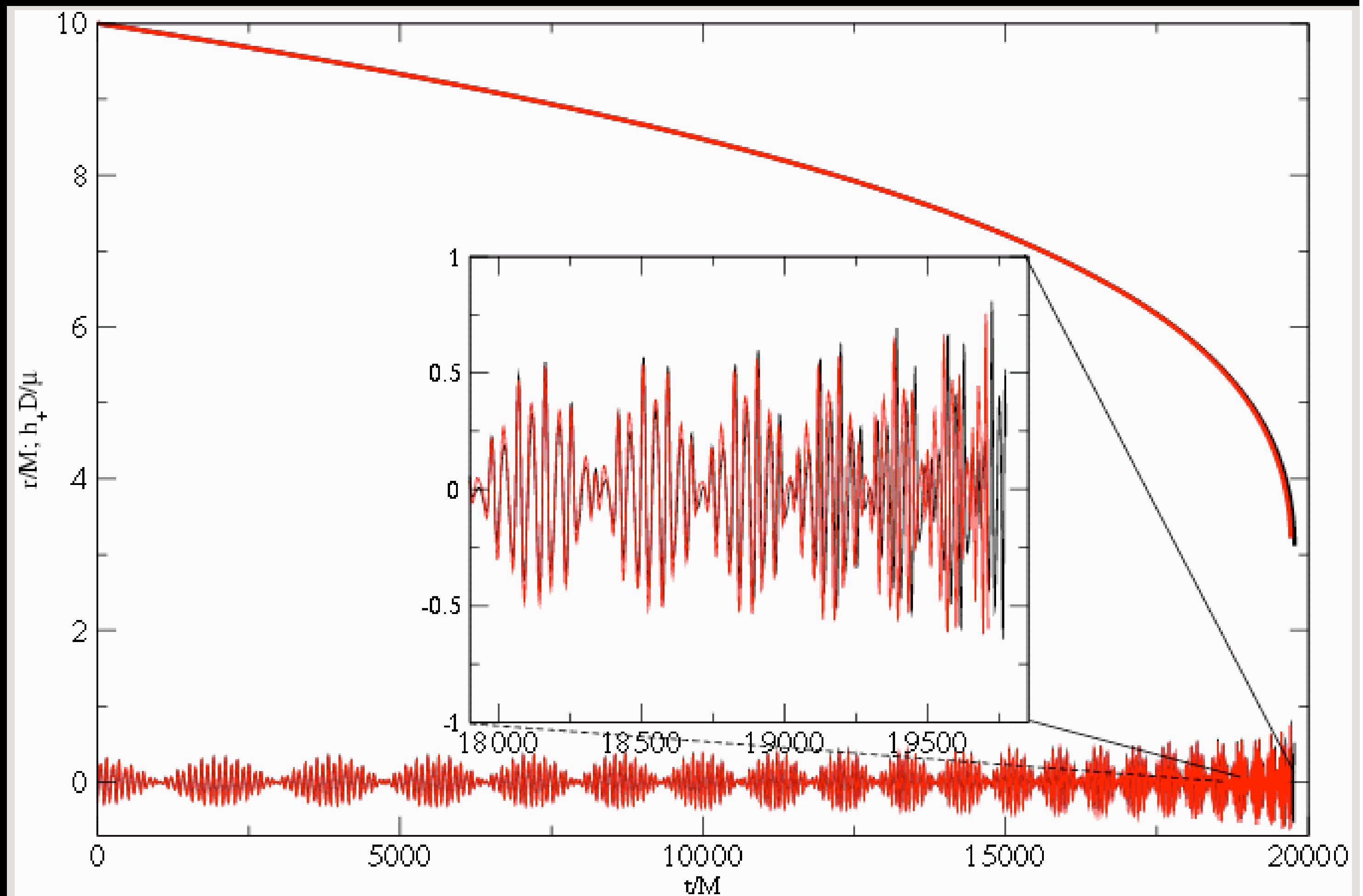
Something this bone-headed isn't supposed to work this well!!!!



Summary; what remains to be done

Adiabatic approximation done in principle … lots of work and computer time needed to get it done in practice.

Need to understand whether adiabatic approach is "good enough" … and need to develop effective data analysis strategies for these waves!

Need to extend analysis to *non* black hole objects: only way we can *test* black hole hypothesis, rather than "just" measure black hole properties.

It is well known that the Kerr solution... provides the unique solution for stationary black holes ... in the universe.

But a confirmation of the metric of the Kerr spacetime (or some aspect of it) cannot even be contemplated in the forseeable future.

Subrahmanyan Chandrasekhar,
The Karl Schwarzschild Lecture, 18 Sept 1986